

Geometry

G.1. Core Content: Logical arguments and proofs

(Logic)

Students formalize the reasoning skills they have developed in previous grades and solidify their understanding of what it means to prove a geometric statement mathematically. In Geometry, students encounter the concept of formal proof built on definitions, axioms, and theorems. They use inductive reasoning to test conjectures about geometric relationships and use deductive reasoning to prove or disprove their conclusions. Students defend their reasoning using precise mathematical language and symbols.

Performance Expectation

Students are expected to:

Explanatory Comments and Examples

G.1.A Distinguish between inductive and deductive reasoning.

Students generate and test conjectures inductively and then prove (or disprove) their conclusions deductively.

Example:

- A student first hypothesizes that the number of degrees in a polygon = $180 \cdot (s - 2)$, where s represents the number of sides, and then proves this is true. When was the student using inductive reasoning? When was s/he using deductive reasoning? Justify your answers.

G.1.B Use inductive reasoning to make conjectures, to test the plausibility of a geometric statement, and to help find a counterexample.

Examples:

- Investigate the relationship among the medians of a triangle using paper folding. Make a conjecture about this relationship.
- Using dynamic geometry software, decide if the following is a plausible conjecture: If segment AM is a median in triangle ABC, then ray AM bisects angle BAC.

G.1.C Use deductive reasoning to prove that a valid geometric statement is true.

Valid proofs may be presented in paragraph, two-column, or flow-chart formats. Proof by contradiction is a form of deductive reasoning.

Example:

- Prove that the diagonals of a rhombus are perpendicular bisectors of each other.

Performance Expectation

Students are expected to:

- G.1.D Write the converse, inverse, and contrapositive of a valid proposition and determine their validity.
- G.1.E Identify errors or gaps in a mathematical argument and develop counterexamples to refute invalid statements about geometric relationships.
- G.1.F Distinguish between definitions and undefined geometric terms and explain the role of definitions, undefined terms, postulates (axioms), and theorems.

Explanatory Comments and Examples

Examples:

- If m and n are odd integers, then the sum of m and n is an even integer. State the converse and determine whether it is valid.
- If a quadrilateral is a rectangle, the diagonals have the same length. State the contrapositive and determine whether it is valid.

Example:

- Identify errors in reasoning in the following proof:
Given $\angle ABC \cong \angle PRQ$, $\overline{AB} \cong \overline{PQ}$, and $\overline{BC} \cong \overline{QR}$, then $\triangle ABC \cong \triangle PQR$ by SAS.

Students sketch *points* and *lines* (undefined terms) and define and sketch representations of other common terms. They use definitions and postulates as they prove theorems throughout geometry. In their work with theorems, they identify the hypothesis and the conclusion and explain the role of each.

Students describe the consequences of changing assumptions or using different definitions for subsequent theorems and logical arguments.

Example:

- There are two definitions of trapezoid that can be found in books or on the web. A trapezoid is either
 - a quadrilateral with exactly one pair of parallel sides or
 - a quadrilateral with at least one pair of parallel sides.

Write some theorems that are true when applied to one definition but not the other, and explain your answer.

Geometry

G.2 Core Content: Lines and Angles

(Geometry/Measurement)

Students study basic properties of parallel and perpendicular lines, their respective slopes, and the properties of the angles formed when parallel lines are intersected by a transversal. They prove related theorems and apply them to solve both mathematical and practical problems.

Performance Expectation

Students are expected to:

Explanatory Comments and Examples

G.2.A Know, prove, and apply theorems about parallel and perpendicular lines.

Students should be able to summarize and explain basic theorems. They are not expected to recite lists of theorems, but they should know the conclusion of a theorem when given its hypothesis.

Examples:

- Prove that a point on the perpendicular bisector of a line segment is equidistant from the ends of the line segment.
- If each of two lines is perpendicular to a given line, what is the relationship between the two lines? How do you know?

G.2.B Know, prove, and apply theorems about angles, including angles that arise from parallel lines intersected by a transversal.

Example:

- Prove that if two parallel lines are cut by a transversal, then alternate-interior angles are equal.
- Take two parallel lines l and m , with (distinct) points A and B on l and C and D on m . If \overline{AC} intersects \overline{BD} at point E , prove that $\triangle ABE \approx \triangle CDE$.

G.2.C Explain and perform basic compass and straightedge constructions related to parallel and perpendicular lines.

Constructions using circles and lines with dynamic geometry software (i.e., virtual compass and straightedge) are equivalent to paper and pencil constructions.

Example:

- Construct and mathematically justify the steps to:
 - Bisect a line segment.
 - Drop a perpendicular from a point to a line.
 - Construct a line through a point that is parallel to another line.

G.2.D Describe the intersections of lines in the plane and in space, of lines and planes, and of planes in space.

Example:

- Describe all the ways that three planes can intersect in space.

Geometry

G.3. Core Content: Two- and three-dimensional figures

(Geometry/Measurement)

Students know and can prove theorems about two- and three-dimensional geometric figures, both formally and informally. They identify necessary and sufficient conditions for proving congruence, similarity, and properties of figures. Triangles are a primary focus, beginning with general properties of triangles, working with right triangles and special triangles, proving and applying the Pythagorean Theorem and its converse, and applying the basic trigonometric ratios of sine, cosine, and tangent. Students extend their learning to other polygons and the circle, and do some work with three-dimensional figures.

Performance Expectation

Students are expected to:

Explanatory Comments and Examples

G.3.A Know, explain, and apply basic postulates and theorems about triangles and the special lines, line segments, and rays associated with a triangle.

Examples:

- Prove that the sum of the angles of a triangle is 180° .
- Prove and explain theorems about the incenter, circumcenter, orthocenter, and centroid.
- The rural towns of Atwood, Bridgeville, and Carnegie are building a communications tower to serve the needs of all three towns. They want to position the tower so that the distance from each town to the tower is equal. Where should they locate the tower? How far will it be from each town?

G.3.B Determine and prove triangle congruence, triangle similarity, and other properties of triangles.

Students should identify necessary and sufficient conditions for congruence and similarity in triangles, and use these conditions in proofs.

Examples:

- Prove that congruent triangles are similar.
- For a given $\triangle RST$, prove that $\triangle XYZ$, formed by joining the midpoints of the sides of $\triangle RST$, is similar to $\triangle RST$.
- Show that not all SSA triangles are congruent.

G.3.C Use the properties of special right triangles (30° – 60° – 90° and 45° – 45° – 90°) to solve problems.

Examples:

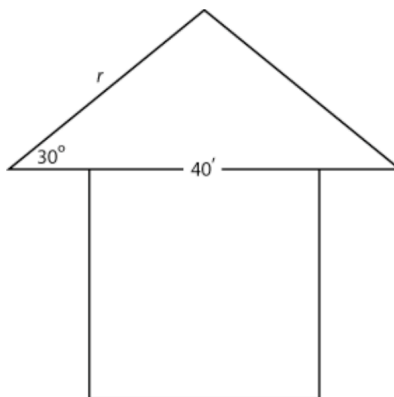
- Determine the length of the altitude of an equilateral triangle whose side lengths measure 5 units.
- If one leg of a right triangle has length 5 and the adjacent angle is 30° , what is the length of the other leg and the hypotenuse?

Performance Expectation

Students are expected to:

Explanatory Comments and Examples

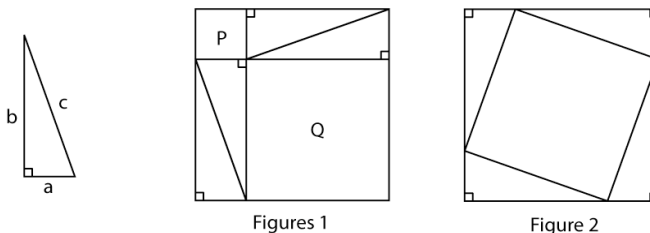
- If one leg of a $45^\circ\text{--}45^\circ\text{--}90^\circ$ triangle has length 5, what is the length of the hypotenuse?
- The pitch of a symmetrical roof on a house 40 feet wide is 30° . What is the length of the rafter, r , exactly and approximately?



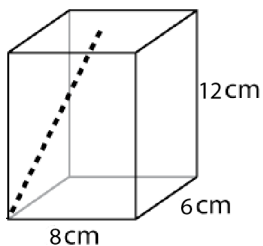
G.3.D Know, prove, and apply the Pythagorean Theorem and its converse.

Examples:

- Consider any right triangle with legs a and b and hypotenuse c . The right triangle is used to create Figures 1 and 2. Explain how these figures constitute a visual representation of a proof of the Pythagorean Theorem.



- A juice box is 6 cm by 8 cm by 12 cm. A straw is inserted into a hole in the center of the top of the box. The straw must stick out 2 cm so you can drink from it. If the straw must be long enough to touch each bottom corner of the box, what is the minimum length the straw must be? (Assume the diameter of the straw is 0 for the mathematical model.)



Performance Expectation

Students are expected to:

Explanatory Comments and Examples

- In $\triangle ABC$, with right angle at C , draw the altitude \overline{CD} from C to \overline{AB} . Name all similar triangles in the diagram. Use these similar triangles to prove the Pythagorean Theorem.
 - Apply the Pythagorean Theorem to derive the distance formula in the (x, y) plane.
- G.3.E Solve problems involving the basic trigonometric ratios of sine, cosine, and tangent.
- Examples:
- A 12-foot ladder leans against a wall to form a 63° angle with the ground. How many feet above the ground is the point on the wall at which the ladder is resting?
 - Use the Pythagorean Theorem to establish that $\sin^2 \theta + \cos^2 \theta = 1$ for θ between 0° and 90° .
- G.3.F Know, prove, and apply basic theorems about parallelograms.
- Properties may include those that address symmetry and properties of angles, diagonals, and angle sums. Students may use inductive and deductive reasoning and counterexamples.
- Examples:
- Are opposite sides of a parallelogram always congruent? Why or why not?
 - Are opposite angles of a parallelogram always congruent? Why or why not?
 - Prove that the diagonals of a parallelogram bisect each other.
 - Explain why if the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
 - Prove that the diagonals of a rectangle are congruent. Is this true for any parallelogram? Justify your reasoning.

Performance Expectation

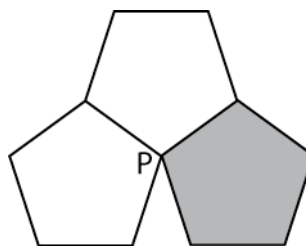
Students are expected to:

G.3.G Know, prove, and apply theorems about properties of quadrilaterals and other polygons.

Explanatory Comments and Examples

Examples:

- What is the length of the apothem of a regular hexagon with side length 8? What is the area of the hexagon?
- If the shaded pentagon were removed, it could be replaced by a regular n -sided polygon that would exactly fill the remaining space. Find the number of sides, n , of a replacement polygon that makes the three polygons fit perfectly.



G.3.H Know, prove, and apply basic theorems relating circles to tangents, chords, radii, secants, and inscribed angles.

Examples:

- Given a line tangent to a circle, know and explain that the line is perpendicular to the radius drawn to the point of tangency.
- Prove that two chords equally distant from the center of a circle are congruent.
- Prove that if one side of a triangle inscribed in a circle is a diameter, then the triangle is a right triangle.
- Prove that if a radius of a circle is perpendicular to a chord of a circle, then the radius bisects the chord.

G.3.I Explain and perform constructions related to the circle.

Students perform constructions using straightedge and compass, paper folding, and dynamic geometry software. What is important is that students understand the mathematics and are able to justify each step in a construction.

Example:

- In each case, explain why the constructions work:
 - a. Construct the center of a circle from two chords.
 - b. Construct a circumscribed circle for a triangle.
 - c. Inscribe a circle in a triangle.

Performance Expectation

Students are expected to:

G.3.J Describe prisms, pyramids, parallelepipeds, tetrahedra, and regular polyhedra in terms of their faces, edges, vertices, and properties.

G.3.K Analyze cross-sections of cubes, prisms, pyramids, and spheres and identify the resulting shapes.

Explanatory Comments and Examples

Examples:

- Given the number of faces of a regular polyhedron, derive a formula for the number of vertices.
- Describe symmetries of three-dimensional polyhedra and their two-dimensional faces.
- Describe the lateral faces that are required for a pyramid to be a right pyramid with a regular base. Describe the lateral faces required for an oblique pyramid that has a regular base.

Examples:

- Start with a regular tetrahedron with edges of unit length 1. Find the plane that divides it into two congruent pieces and whose intersection with the tetrahedron is a square. Find the area of the square. (Requires no pencil or paper.)
- Start with a cube with edges of unit length 1. Find the plane that divides it into two congruent pieces and whose intersection with the cube is a regular hexagon. Find the area of the hexagon.
- Start with a cube with edges of unit length 1. Find the plane that divides it into two congruent pieces and whose intersection with the cube is a rectangle that is not a face and contains four of the vertices. Find the area of the rectangle.
- Which has the larger area, the above rectangle or the above hexagon?

Geometry

G.4. Core Content: Geometry in the coordinate plane (Geometry/Measurement, Algebra)

Students make connections between geometry and algebra by studying geometric properties and attributes that can be represented on the coordinate plane. They use the coordinate plane to represent situations that are both purely mathematical and that arise in applied contexts. In this way, they use the power of algebra to solve problems about shapes and space.

Performance Expectation

Students are expected to:

Explanatory Comments and Examples

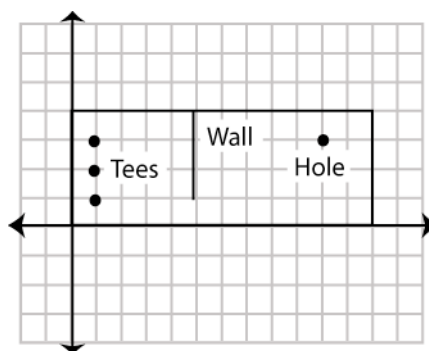
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| <p>G.4.A Determine the equation of a line in the coordinate plane that is described geometrically, including a line through two given points, a line through a given point parallel to a given line, and a line through a given point perpendicular to a given line.</p> | <p>Examples:</p> <ul style="list-style-type: none">• Write an equation for the perpendicular bisector of a given line segment.• Determine the equation of a line through the points (5, 3) and (5, -2).• Prove that the slopes of perpendicular lines are negative inverses of each other. |
| <p>G.4.B Determine the coordinates of a point that is described geometrically.</p> | <p>Examples:</p> <ul style="list-style-type: none">• Determine the coordinates for the midpoint of a given line segment.• Given the coordinates of three vertices of a parallelogram, determine all possible coordinates for the fourth vertex.• Given the coordinates for the vertices of a triangle, find the coordinates for the center of the circumscribed circle and the length of its radius. |
| <p>G.4.C Verify and apply properties of triangles and quadrilaterals in the coordinate plane.</p> | <p>Examples:</p> <ul style="list-style-type: none">• Given four points in a coordinate plane that are the vertices of a quadrilateral, determine whether the quadrilateral is a rhombus, a square, a rectangle, a parallelogram, or none of these. Name all the classifications that apply.• Given a parallelogram on a coordinate plane, verify that the diagonals bisect each other.• Given the line with y-intercept 4 and x-intercept 3, find the area of a square that has one corner on the origin and the opposite corner on the line described. |

Performance Expectation

Students are expected to:

Explanatory Comments and Examples

- Below is a diagram of a miniature golf hole as drawn on a coordinate grid. The dimensions of the golf hole are 4 feet by 12 feet. Players must start their ball from one of the three tee positions, located at (1, 1), (1, 2), or (1, 3). The hole is located at (10, 3). A wall separates the tees from the hole. At which tee should the ball be placed to create the shortest "hole-in-one" path? Sketch the intended path of the ball, find the distance the ball will travel, and describe your reasoning. (Assume the diameters of the golf ball and the hole are 0 for the mathematical model.)



G.4.D Determine the equation of a circle that is described geometrically in the coordinate plane and, given equations for a circle and a line, determine the coordinates of their intersection(s).

Examples:

- Write an equation for a circle with a radius of 2 units and center at (1, 3).
- Given the circle $x^2 + y^2 = 4$ and the line $y = x$, find the points of intersection.
- Write an equation for a circle given a line segment as a diameter.
- Write an equation for a circle determined by a given center and tangent line.

Geometry

G.5. Core Content: Geometric transformations

(Geometry/Measurement)

Students continue their study of geometric transformations, focusing on the effect of such transformations and the composition of transformations on the attributes of geometric figures. They study techniques for establishing congruence and similarity by means of transformations.

Performance Expectation

Students are expected to:

Explanatory Comments and Examples

<p>G.5.A Sketch results of transformations and compositions of transformations for a given two-dimensional figure on the coordinate plane, and describe the rule(s) for performing translations or for performing reflections about the coordinate axes or the line $y = x$.</p>	<p>Transformations include translations, rotations, reflections, and dilations.</p> <p>Example:</p> <ul style="list-style-type: none">Line m is described by the equation $y = 2x + 3$. Graph line m and reflect line m across the line $y = x$. Determine the equation of the image of the reflection. Describe the relationship between the line and its image.
<p>G.5.B Determine and apply properties of transformations.</p>	<p>Students make and test conjectures about compositions of transformations and inverses of transformations, the commutativity and associativity of transformations, and the congruence and similarity of two-dimensional figures under various transformations.</p> <p>Examples:</p> <ul style="list-style-type: none">Identify transformations (alone or in composition) that preserve congruence.Determine whether the composition of two reflections of a line is commutative.Determine whether the composition of two rotations about the same point of rotation is commutative.Find a rotation that is equivalent to the composition of two reflections over intersecting lines.Find the inverse of a given transformation.
<p>G.5.C Given two congruent or similar figures in a coordinate plane, describe a composition of translations, reflections, rotations, and dilations that superimposes one figure on the other.</p>	<p>Examples:</p> <ul style="list-style-type: none">Find a sequence of transformations that superimposes the segment with endpoints $(0, 0)$ and $(2, 1)$ on the segment with endpoints $(4, 2)$ and $(3, 0)$.Find a sequence of transformations that superimposes the triangle with vertices $(0, 0)$, $(1, 1)$, and $(2, 0)$ on the triangle with vertices $(0, 1)$, $(2, -1)$, and $(0, -3)$.

Performance Expectation

Students are expected to:

G.5.D Describe the symmetries of two-dimensional figures and describe transformations, including reflections across a line and rotations about a point.

Explanatory Comments and Examples

Although the expectation only addresses two-dimensional figures, classroom activities can easily extend to three-dimensional figures. Students can also describe the symmetries, reflections across a plane, and rotations about a line for three-dimensional figures.

Geometry

G.6. Additional Key Content

(Measurement)

Students extend and formalize their work with geometric formulas for perimeter, area, surface area, and volume of two- and three-dimensional figures, focusing on mathematical derivations of these formulas and their applications in complex problems. They use properties of geometry and measurement to solve problems in purely mathematical as well as applied contexts. Students understand the role of units in measurement and apply what they know to solve problems involving derived measures like speed or density. They understand that all measurement is approximate and specify precision in measurement problems.

Performance Expectation

Students are expected to:

Explanatory Comments and Examples

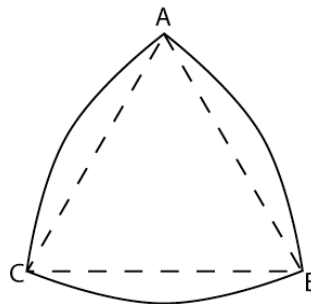
- G.6.A Derive and apply formulas for arc length and area of a sector of a circle.

Example:

- Find the area and perimeter of the Reuleaux triangle below.

The Reuleaux triangle is constructed with three arcs. The center of each arc is located at the vertex of an equilateral triangle. Each arc extends between the two opposite vertices of the equilateral triangle.

The figure below is a Reuleaux triangle that circumscribes equilateral triangle ABC . $\triangle ABC$ has side length of 5 inches. \widehat{AB} has center C , \widehat{BC} has center A , and \widehat{CA} has center B , and all three arcs have the same radius equal to the length of the sides of the triangle.



- G.6.B Analyze distance and angle measures on a sphere and apply these measurements to the geometry of the earth.

Examples:

- Use a piece of string to measure the distance between two points on a ball or globe; verify that the string lies on an arc of a great circle.
- On a globe, show with examples why airlines use polar routes instead of flying due east from Seattle to Paris.
- Show that the sum of the angles of a triangle on a sphere is greater than 180 degrees.

Performance Expectation

Students are expected to:

G.6.C Apply formulas for surface area and volume of three-dimensional figures to solve problems.

G.6.D Predict and verify the effect that changing one, two, or three linear dimensions has on perimeter, area, volume, or surface area of two- and three-dimensional figures.

G.6.E Use different degrees of precision in measurement, explain the reason for using a certain degree of precision, and apply estimation strategies to obtain reasonable measurements with appropriate precision for a given purpose.

Explanatory Comments and Examples

Problems include those that are purely mathematical as well as those that arise in applied contexts.

Three-dimensional figures include right and oblique prisms, pyramids, cylinders, cones, spheres, and composite three-dimensional figures.

Example:

- As Pam scooped ice cream into a cone, she began to formulate a geometry problem in her mind. If the ice cream was perfectly spherical with diameter 2.25" and sat on a geometric cone that also had diameter 2.25" and was 4.5" tall, would the cone hold all the ice cream as it melted (without her eating any of it)? She figured the melted ice cream would have the same volume as the unmelted ice cream.

Find the solution to Pam's problem and justify your reasoning.

- A rectangle is 5 inches by 10 inches. Find the volume of a cylinder that is generated by rotating the rectangle about the 10-inch side.

The emphasis in high school should be on verifying the relationships between length, area, and volume and on making predictions using algebraic methods.

Example:

- What happens to the volume of a rectangular prism if four parallel edges are doubled in length?
- The ratio of a pair of corresponding sides in two similar triangles is 5:3. The area of the smaller triangle is 108 in^2 . What is the area of the larger triangle?

Example:

- The U.S. Census Bureau reported a national population of 299,894,924 on its Population Clock in mid-October of 2006. One can say that the U.S. population is 3 hundred million (3×10^8) and be precise to one digit. Although the population had surpassed 3 hundred million by the end of that month, explain why 3×10^8 remained precise to one digit.

Performance Expectation

Students are expected to:

G.6.F Solve problems involving measurement conversions within and between systems, including those involving derived units, and analyze solutions in terms of reasonableness of solutions and appropriate units.

Explanatory Comments and Examples

This performance expectation is intended to build on students' knowledge of proportional relationships. Students should understand the relationship between scale factors and their inverses as they relate to choices about when to multiply and when to divide in converting measurements.

Derived units include those that measure speed, density, flow rates, population density, etc.

Example:

- A digital camera takes pictures that are 3.2 megabytes in size. If the pictures are stored on a 1-gigabyte card, how many pictures can be taken before the card is full?

Geometry

G.7. Core Processes: Reasoning, problem solving, and communication

Students formalize the development of reasoning in Geometry as they become more sophisticated in their ability to reason inductively and begin to use deductive reasoning in formal proofs. They extend the problem-solving practices developed in earlier grades and apply them to more challenging problems, including problems related to mathematical and applied situations. Students use a coherent problem-solving process in which they analyze the situation to determine the question(s) to be answered, synthesize given information, and identify implicit and explicit assumptions that have been made. They examine their solution(s) to determine reasonableness, accuracy, and meaning in the context of the original problem. They use correct mathematical language, terms, symbols, and conventions as they address problems in Geometry and provide descriptions and justifications of solution processes. The mathematical thinking, reasoning, and problem-solving processes students learn in high school mathematics can be used throughout their lives as they deal with a world in which an increasing amount of information is presented in quantitative ways, and more and more occupations and fields of study rely on mathematics.

Performance Expectation

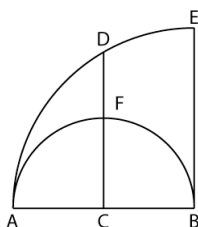
Students are expected to:

- G.7.A Analyze a problem situation and represent it mathematically.
- G.7.B Select and apply strategies to solve problems.
- G.7.C Evaluate a solution for reasonableness, verify its accuracy, and interpret the solution in the context of the original problem.
- G.7.D Generalize a solution strategy for a single problem to a class of related problems, and apply a strategy for a class of related problems to solve specific problems.
- G.7.E Read and interpret diagrams, graphs, and text containing the symbols, language, and conventions of mathematics.
- G.7.F Summarize mathematical ideas with precision and efficiency for a given audience and purpose.

Explanatory Comments and Examples

Examples:

- \overline{AB} is the diameter of the semicircle and the radius of the quarter circle shown in the figure below. \overline{DC} is the perpendicular bisector of \overline{AB} .



Imagine all of the triangles formed by \overline{AB} and any arbitrary point lying in the region bounded by \overline{AC} , \overline{CD} , and \overline{AD} , seen in bold below.



Performance Expectation

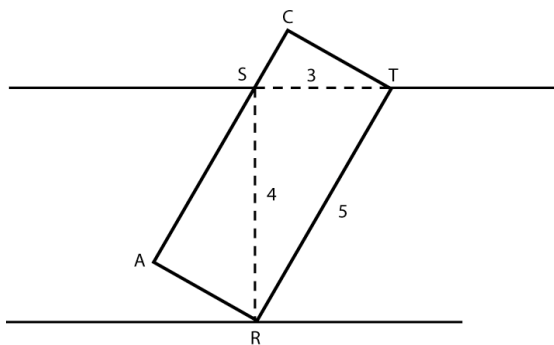
Students are expected to:

- G.7.G Synthesize information to draw conclusions and evaluate the arguments and conclusions of others.
- G.7.H Use inductive reasoning to make conjectures, and use deductive reasoning to prove or disprove conjectures.

Explanatory Comments and Examples

Use inductive reasoning to make conjectures about what types of triangles are formed based upon the region where the third vertex is located. Use deductive reasoning to verify your conjectures.

- Rectangular cartons that are 5 feet long need to be placed in a storeroom that is located at the end of a hallway. The walls of the hallway are parallel. The door into the hallway is 3 feet wide and the width of the hallway is 4 feet. The cartons must be carried face up. They may not be tilted. Investigate the width and carton top area that will fit through the doorway.



Generalize your results for a hallway opening of x feet and a hallway width of y feet if the maximum carton dimensions are c feet long and $x^2 + y^2 = c^2$.