

Algebra 2

A2.1. Core Content: Solving Problems

The first core content area highlights the type of problems students will be able to solve by the end of Algebra 1I, as they extend their ability to solve problems with additional functions and equations. When presented with a word problem, students are able to determine which function or equation models the problem and use that information to solve the problem. They build on what they learned in Algebra I about linear and quadratic functions and are able to solve more complex problems. Additionally, students learn to solve problems modeled by exponential and logarithmic functions, systems of equations and inequalities, inverse variations, and combinations and permutations. Turning word problems into equations that can be solved is a skill students hone throughout Algebra II and subsequent mathematics courses.

Performance Expectation

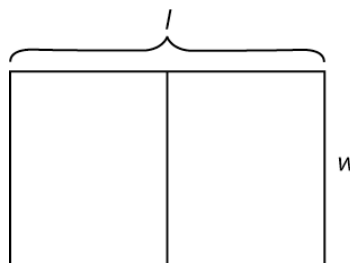
Students are expected to:

A2.1.A Select and justify functions and equations to model and solve problems.

Explanatory Comments and Examples

Examples:

- A manufacturer wants to design a cylindrical soda can that will hold 500 milliliters (mL) of soda. The manufacturer's research has determined that an optimal can height is between 10 and 15 centimeters. Find a function for the radius in terms of the height, and use it to find the possible range of radius measurements in centimeters. Explain your reasoning.
- Dawson wants to make a horse corral by creating a rectangle that is divided into 2 parts, similar to the following diagram. He has a 1200-foot roll of fencing to do the job.
 - What are the dimensions of the enclosure with the largest total area?
 - What function or equation best models this situation?



Performance Expectation

Students are expected to:

A2.1.B Solve problems that can be represented by systems of equations and inequalities.

A2.1.C Solve problems that can be represented by quadratic functions, equations, and inequalities.

Explanatory Comments and Examples

Examples:

- Mr. Smith uses the following formula to calculate students' final grades in his Algebra 1I class: $0.4E + 0.6T = C$, where E represents the score on the final exam, and T represents the average score of all tests given during the grading period. All tests and the final exam are worth a maximum of 100 points. The minimum passing score on tests, the final exam, and the course is 60.

Determine the inequalities that describe the following situation and sketch a system of graphs to illustrate it. When necessary, round scores to the nearest tenth.

- Is it possible for a student to have a failing test score average (i.e., $T < 60$ points) and still pass the course?
- If you answered “yes,” what is the minimum test score average a student can have and still pass the course? What final exam score is needed to pass the course with a minimum test score average?
- A student has a particular test score average. How can (s)he figure out the minimum final exam score needed to pass the course?
- Data derived from an experiment seems to be parabolic when plotted on a coordinate grid. Three observed data points are (2, 10), (3, 8), and (4, 4). Write a quadratic equation that passes through the points.

In addition to solving area and velocity problems by factoring and applying the quadratic formula to the quadratic equation, students use the vertex form of the equation to solve problems about maximums, minimums, and symmetry.

Examples:

- The Gateway Arch in St. Louis has a special shape called a catenary, which looks a lot like a parabola. It has a base width of 600 feet and is 630 feet high. Which is taller, this catenary arch or a parabolic arch that has the same base width but has a height of 450 feet at a point 150 feet from one of the pillars? What is the height of the parabolic arch?

Performance Expectation

Students are expected to:

A2.1.D Solve problems that can be represented by exponential and logarithmic functions and equations.

A2.1.E Solve problems that can be represented by inverse variations of

the forms $f(x) = \frac{a}{x} + b$,

$f(x) = \frac{a}{x^2} + b$, and $f(x) = \frac{a}{(bx + c)}$.

Explanatory Comments and Examples

- Fireworks are launched upward from the ground with an initial velocity of 160 feet per second. The formula for vertical motion is $h(t) = 0.5at^2 + vt + s$, where the gravitational constant, a , is -32 feet per square second, v represents the initial velocity, and s represents the initial height. Time t is measured in seconds, and height h is measured in feet.

For the ultimate effect, the fireworks must explode after they reach the maximum height. For the safety of the crowd, they must explode at least 256 ft. above the ground. The fuses must be set for the appropriate time interval that allows the fireworks to reach this height. What range of times, starting from initial launch and ending with fireworks explosion, meets these conditions?

Examples:

- If you need \$15,000 in 4 years to start college, how much money would you need to invest now? Assume an annual interest rate of 4% compounded monthly for 48 months.
- The half-life of a certain radioactive substance is 65 days. If there are 4.7 grams initially present, how long will it take for there to be less than 1 gram of the substance remaining?

Examples:

- At the You're Toast, Dude! toaster company, the weekly cost to run the factory is \$1400, and the cost of producing each toaster is an additional \$4 per toaster.
 - Find a function to represent the weekly cost in dollars, $C(x)$, of producing x toasters. Assume either unlimited production is possible or set a maximum per week.
 - Find a function to represent the total production cost per toaster for a week.
 - How many toasters must be produced within a week to have a total production cost per toaster of \$8?
- A person's weight varies inversely as the square of his distance from the center of the earth. Assume the radius of the earth is 4000 miles. How much would a 200-pound man weigh
 - 1000 miles above the surface of the earth?
 - 2000 miles above the surface of the earth?

Performance Expectation

Students are expected to:

A2.1.F Solve problems involving combinations and permutations.

Explanatory Comments and Examples

Example:

- The company Ali works for allows her to invest in her choice of 10 different mutual funds, 6 of which grew by at least 5% over the last year. Ali randomly selected 4 of the 10 funds in which to invest. What is the probability that 3 of Ali's funds grew by 5%?
- Four points (A , B , C , and D) lie on one straight line, n , and five points (E , F , G , H , and J) lie on another straight line, m , that is parallel to n . What is the probability that three points, selected at random, will form a triangle?

Algebra 2

A2.2. Core Content: Numbers, expressions, and operations (Numbers, Operations, Algebra)

Students extend their understanding of number systems to include complex numbers, which they will see as solutions for quadratic equations. They grow more proficient in their use of algebraic techniques as they continue to use variables and expressions to solve problems. As problems become more sophisticated and the level of mathematics increases, so does the complexity of the symbolic manipulations and computations necessary to solve the problems. Students refine the foundational algebraic skills they need to be successful in subsequent mathematics courses.

Performance Expectation

Students are expected to:

A2.2.A Explain how whole, integer, rational, real, and complex numbers are related, and identify the number system(s) within which a given algebraic equation can be solved.

A2.2.B Use the laws of exponents to simplify and evaluate numeric and algebraic expressions that contain rational exponents.

Explanatory Comments and Examples

Example:

- Within which number system(s) can each of the following be solved? Explain how you know.

— $3x + 2 = 5$

— $x^2 = 1$

— $x^2 = \frac{1}{4}$

— $x^2 = 2$

— $x^2 = -2$

— $\frac{x}{7} = \pi$

Examples:

- Convert the following from a radical to exponential form or visa versa.

— $24^{\frac{1}{3}}$

— $\sqrt[5]{16}$

— $\sqrt{x^2 + 1}$

— $\frac{x^2}{\sqrt{x}}$

- Evaluate $x^{-3/2}$ for $x = 27$

Performance Expectation

Students are expected to:

A2.2.C Add, subtract, multiply, divide, and simplify rational and more general algebraic expressions.

Explanatory Comments and Examples

In the same way that integers were extended to fractions, polynomials are extended to rational expressions. Students must be able to perform the four basic arithmetic operations on more general expressions that involve exponentials.

The binomial theorem is useful when raising expressions to powers, such as $(x + 3)^5$.

Examples:

- $\frac{x+1}{(x+1)^2} - \frac{3x-3}{x^2-1}$
- Divide $\frac{(x+2)^{3/2}}{x+1}$ by $\frac{x+2}{x^2-1}$

Algebra 2

A2.3. Core Content: Quadratic functions and equations

(Algebra)

As students continue to solve quadratic equations and inequalities in Algebra 2, they encounter complex roots for the first time. They learn to translate between forms of quadratic equations, applying the vertex form to evaluate maximum and minimum values and find symmetry of the graph, and they learn to identify which form should be used in a particular situation. This opens up a whole range of new problems students can solve using quadratics. These algebraic skills are applied in subsequent high school mathematics and statistics courses.

Performance Expectation

Students are expected to:

A2.3.A Translate between the standard form of a quadratic function, the vertex form, and the factored form; graph and interpret the meaning of each form.

Explanatory Comments and Examples

Students translate among forms of a quadratic function to convert to one that is appropriate—e.g., vertex form—to solve specific problems.

Students learn about the advantages of the standard form ($f(x) = ax^2 + bx + c$), the vertex form ($f(x) = a(x - h)^2 + d$), and the factored form ($f(x) = a(x - r)(x - s)$). They produce the vertex form by completing the square on the function in standard form, which allows them to see the symmetry of the graph of a quadratic function as well as the maximum or minimum. This opens up a whole range of new problems students can solve using quadratics. Students continue to find the solutions of the equation, which in Algebra 2 can be either real or complex.

Example:

- Find the minimum, the line of symmetry, and the roots for the graphs of each of the following functions:

$$f(x) = x^2 - 4x + 3$$

$$f(x) = x^2 - 4x + 4$$

$$f(x) = x^2 - 4x + 5$$

A2.3.B Determine the number and nature of the roots of a quadratic function.

Students should be able to recognize and interpret the discriminant.

Students should also be familiar with the Fundamental Theorem of Algebra, i.e., that all polynomials, not just quadratics, have roots over the complex numbers. This concept becomes increasingly important as students progress through mathematics.

Example:

- For what values of a does $x^2 - 6x + a$ have 2 real roots, 1 real root, and no real roots?

Performance Expectation

Students are expected to:

A2.3.C Solve quadratic equations and inequalities, including equations with complex roots.

Explanatory Comments and Examples

Students solve equations that are not easily factored by completing the square and by using the quadratic formula.

Examples:

- $x^2 - 10x + 34 = 0$
- $3x^2 + 10 = 4x$
- Wile E. Coyote launches an anvil from 180 feet above the ground at time $t = 0$. The equation that models this situation is given by $h = -16t^2 + 96t + 180$, where t is time measured in seconds and h is height above the ground measured in feet.
 - a. What is a reasonable domain restriction for t in this context?
 - b. Determine the height of the anvil two seconds after it was launched.
 - c. Determine the maximum height obtained by the anvil.
 - d. Determine the time when the anvil is more than 100 feet above ground.
- Farmer Helen wants to build a pigpen. With 100 feet of fence, she wants a rectangular pen with one side being a side of her existing barn. What dimensions should she use for her pigpen in order to have the maximum number of square feet?

Algebra 2

A2.4. Core Content: Exponential and logarithmic functions and equations

(Algebra)

Students extend their understanding of exponential functions from Algebra 1 with an emphasis on inverse functions. This leads to a natural introduction of logarithms and logarithmic functions. They learn to use the basic properties of exponential and logarithmic functions, graphing both types of function to analyze relationships, represent and model problems, and answer questions. Students employ these functions in many practical situations, such as applying exponential functions to determine compound interest and applying logarithmic functions to determine the pH of a liquid.

Performance Expectation

Students are expected to:

A2.4.A Know and use basic properties of exponential and logarithmic functions and the inverse relationship between them.

A2.4.B Graph an exponential function of the form $f(x) = ab^x$ and its inverse logarithmic function.

Explanatory Comments and Examples

Examples:

- Given $f(x) = 4^x$, write an equation for the inverse of this function. Graph the functions on the same coordinate grid.
 - Find $f(-3)$.
 - Evaluate the inverse function at 7.
- Derive the formulas:
 - $\log_b a \cdot \log_a b = 1$
 - $\log_a N = \log_b N \cdot \log_a b$
- Find the exact value of x in:
 - $\log_x 16 = \frac{4}{3}$
 - $\log_3 81 = x$
- Solve for y in terms of x :
 - $\log_a \frac{y}{x} = x$
 - $100 = x \cdot 10^y$

Students expand on the work they did in Algebra 1 to functions of the form $y = ab^x$. Although the concept of inverses is not fully developed until Precalculus, there is an emphasis in Algebra 2 on students recognizing the inverse relationship between exponential and logarithmic functions and how this is reflected in the shapes of the graphs.

Example:

- Find the equation for the inverse function of $y = 3^x$. Graph both functions. What characteristics of each of the graphs indicate they are inverse functions?

Performance Expectation

Students are expected to:

A2.4.C Solve exponential and logarithmic equations.

Explanatory Comments and Examples

Example:

- A recommended adult dosage of the cold medication NoMoreFlu is 16 mL. NoMoreFlu causes drowsiness when there are more than 4 mL in one's system, making it unsafe to drive, operate machinery, etc. The manufacturer wants to print a warning label telling people how long they should wait after taking NoMoreFlu for the drowsiness to pass. If the typical metabolic rate is such that one quarter of the NoMoreFlu is lost every four hours, and a person takes the full dosage, how long should adults wait after taking NoMoreFlu to ensure that there will be
 - Less than 4 mL of NoMoreFlu in their system?
 - Less than 1 mL in their system?
 - Less than 0.1 mL in their system?
- Solve for x in $256 = 2^{x^2-1}$.
- Solve for x in $\log_5(x - 4) = 3$

Algebra 2

A2.5. Core Content: Additional functions and equations

(Algebra)

Students learn about additional classes of functions including square root, cubic, logarithmic, and those involving inverse variation. Students plot points and sketch graphs to represent these functions and use algebraic techniques to solve related equations. In addition to studying the defining characteristics of each of these classes of functions, students gain the ability to construct new functions algebraically and using transformations. These extended skills and techniques serve as the foundation for further study and analysis of functions in subsequent mathematics courses.

Performance Expectation

Students are expected to:

A2.5.A Construct new functions using the transformations $f(x - h)$, $f(x) + k$, $cf(x)$, and by adding and subtracting functions, and describe the effect on the original graph(s).

Explanatory Comments and Examples

Students perform simple transformations on functions, including those that contain the absolute value of expressions, quadratic expressions, square root expressions, and exponential expressions, to make new functions.

Examples:

- What sequence of transformations changes $f(x) = x^2$ to $g(x) = -5(x - 3)^2 + 2$?
- Carly decides to earn extra money by making glass bead bracelets. She purchases tools for \$40. Elastic bead cord for each bracelet costs \$0.10. Glass beads come in packs of 10 beads, and one pack has enough beads to make one bracelet. Base price for the beads is \$2.00 per pack. For each of the first 100 packs she buys, she gets \$0.01 off each of the packs. (For example, if she purchases three packs, each pack costs \$1.97 instead of \$2.00.) Carly plans to sell each bracelet for \$4.00. Assume Carly will make a maximum of 100 bracelets.
 - Find a function $C(b)$ that describes Carly's costs.
 - Find a function $R(b)$ that describes Carly's revenue.Carly's profit is described by $P(b) = R(b) - C(b)$.
 - Find $P(b)$.
 - What is the minimum number of bracelets that Carly must sell in order to make a profit?
 - To make a profit of \$100?

Performance Expectation

Students are expected to:

A2.5.B Plot points, sketch, and describe the graphs of functions of the form $f(x) = a\sqrt{x - c} + d$, and solve related equations.

A2.5.C Plot points, sketch, and describe the graphs of functions of the form

$$f(x) = \frac{a}{x} + b, \quad f(x) = \frac{a}{x^2} + b,$$

and $f(x) = \frac{a}{(bx + c)}$, and solve related equations.

A2.5.D Plot points, sketch, and describe the graphs of cubic polynomial functions of the form $f(x) = ax^3 + d$ as an example of higher order polynomials and solve related equations.

Explanatory Comments and Examples

Students solve algebraic equations that involve the square root of a linear expression over the real numbers. Students should be able to identify extraneous solutions and explain how they arose.

Students should view the function $g(x) = \sqrt{x}$ as the inverse function of $f(x) = x^2$, recognizing that the functions have different domains for x greater than or equal to 0.

Example:

- Analyze the following equations and tell what you know about the solutions. Then solve the equations.

$$\text{— } 2\sqrt{x+5} = 7$$

$$\text{— } \sqrt{5x-6} = -2$$

$$\text{— } \sqrt{2x+15} = x$$

$$\text{— } \sqrt{2x-5} = x+7$$

Examples:

- Sketch the graphs of the four functions $f(x) = \frac{a}{x^2} + b$ when $a = 4$ and 8 and $b = 0$ and 1 .
- Sketch the graphs of the four functions $f(x) = \frac{4}{(bx + c)}$ when $b = 1$ and 4 and $c = 2$ and 3 .

Examples:

- Solve for x in $60 = -2x^3 + 6$.

Algebra 2

A2.6. Core Content: Probability, data, and distributions

(Data/Statistics/Probability)

Students formalize their study of probability, computing both combinations and permutations to calculate the likelihood of an outcome in uncertain circumstances and applying the binomial theorem to solve problems. They extend their use of statistics to graph bivariate data and analyze its shape to make predictions. They calculate and interpret measures of variability, confidence intervals, and margins of error for population proportions. Dual goals underlie the content in the section: students prepare for the further study of statistics and become thoughtful consumers of data.

Performance Expectation

Students are expected to:

A2.6.A Apply the fundamental counting principle and the ideas of order and replacement to calculate probabilities in situations arising from two-stage experiments (compound events).

A2.6.B Given a finite sample space consisting of equally likely outcomes and containing events A and B, determine whether A and B are independent or dependent, and find the conditional probability of A given B.

A2.6.C Compute permutations and combinations, and use the results to calculate probabilities.

A2.6.D Apply the binomial theorem to solve problems involving probability.

Explanatory Comments and Examples

Example:

- What is the probability of drawing a heart from a standard deck of cards on a second draw, given that a heart was drawn on the first draw and not replaced?

Example:

- Two friends, Abby and Ben, are among five students being considered for three student council positions. If each of the five students has an equal likelihood of being selected, what is the probability that Abby and Ben will both be selected?

The binomial theorem is also applied when computing with polynomials.

Examples:

- Use Pascal's triangle and the binomial theorem to find the number of ways six objects can be selected four at a time.
- In a survey, 33% of adults reported that they preferred to get the news from newspapers rather than television. If you survey 5 people, what is the probability of getting exactly 2 people who say they prefer news from the newspaper?
 - Write an equation that can be used to solve the problem.
 - Create a histogram of the binomial distribution of the probability of getting 0 through 5 responders saying they prefer the newspaper.

Performance Expectation

Students are expected to:

A2.6.E Determine if a bivariate data set can be better modeled with an exponential or a quadratic function and use the model to make predictions.

A2.6.F Calculate and interpret measures of variability and standard deviation and use these measures and the characteristics of the normal distribution to describe and compare data sets.

A2.6.G Calculate and interpret margin of error and confidence intervals for population proportions.

Explanatory Comments and Examples

In high school, determining a formula for a curve of best fit requires a graphing calculator or similar technological tool.

Students should be able to identify unimodality, symmetry, standard deviation, spread, and the shape of a data curve to determine whether the curve could reasonably be approximated by a normal distribution.

Given formulas, student should be able to calculate the standard deviation for a small data set, but calculators ought to be used if there are very many points in the data set. It is important that students be able to describe the characteristics of the normal distribution and identify common examples of data that are and are not reasonably modeled by it. Common examples of distributions that are approximately normal include physical performance measurements (e.g., weightlifting, timed runs), heights, and weights.

Apply the Empirical Rule (68–95–99.7 Rule) to approximate the percentage of the population meeting certain criteria in a normal distribution.

Example:

- Which is more likely to be affected by an outlier in a set of data, the interquartile range or the standard deviation?

Students will use technology based on the complexity of the situation.

Students use confidence intervals to critique various methods of statistical experimental design, data collection, and data presentation used to investigate important problems, including those reported in public studies.

Example:

- In 2007, 400 of the 500 10th graders in Local High School passed the WASL. In 2008, 375 of the 480 10th graders passed the test. The Local Gazette headline read “10th Grade WASL Scores Decline in 2008!” In response, the Superintendent of Local School District wrote a letter to the editor claiming that, in fact, WASL performance was not significantly lower in 2008 than it was in 2007. Who is correct, the Local Gazette or the Superintendent?

Performance Expectation

Students are expected to:

Explanatory Comments and Examples

Use mathematics to find the margin of error to justify your conclusion. (Formula for the margin of

error (E): $E = z_c \sqrt{\frac{p(1-p)}{n}}$; $z_{95} = 1.96$, where n is

the sample size, p is the proportion of the sample with the trait of interest, c is the confidence level, and z_c is the multiplier for the specified confidence interval.)

Algebra 2

A2.7. Additional Key Content

(Algebra)

Students study two important topics here. First, they extend their ability to solve systems of two equations in two variables to solving systems of three equations in three variables, which leads to the full development of matrices in Precalculus. Second, they formalize their work with series as they learn to find the terms and partial sums of arithmetic series and the terms and partial and infinite sums of geometric series. This conceptual understanding of series lays an important foundation for understanding calculus.

Performance Expectation

Students are expected to:

A2.7.A Solve systems of three equations with three variables.

A2.7.B Find the terms and partial sums of arithmetic and geometric series and the infinite sum for geometric series.

Explanatory Comments and Examples

Students solve systems of equations using algebraic and numeric methods.

Examples:

- Jill, Ann, and Stan are to inherit \$20,000. Stan is to get twice as much as Jill, and Ann is to get twice as much as Stan. How much does each get?
- Solve the following system of equations.

$$2x - y - z = 7$$

$$3x + 5y + z = -10$$

$$4x - 3y + 2z = 4$$

Students build on the knowledge gained in Algebra 1 to find specific terms in a sequence and to express arithmetic and geometric sequences in both explicit and recursive forms.

Examples:

- A ball is dropped from a height of 10 meters. Each time it hits the ground, it rebounds $\frac{3}{4}$ of the distance it has fallen. What is the total sum of the distances it falls and rebounds before coming to rest?
- Show that the sum of the first 10 terms of the geometric series $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots$ is twice the sum of the first 10 terms of the geometric series $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \dots$

Algebra 2

A2.8. Core Processes: Reasoning, problem solving, and communication

Students formalize the development of reasoning at high school as they use algebra and the properties of number systems to develop valid mathematical arguments, make and prove conjectures, and find counterexamples to refute false statements using correct mathematical language, terms, and symbols in all situations. They extend the problem-solving practices developed in earlier grades and apply them to more challenging problems, including problems related to mathematical and applied situations. Students formalize a coherent problem-solving process in which they analyze the situation to determine the question(s) to be answered, synthesize given information, and identify implicit and explicit assumptions that have been made. They examine their solution(s) to determine reasonableness, accuracy, and meaning in the context of the original problem. The mathematical thinking, reasoning, and problem-solving processes students learn in high school mathematics can be used throughout their lives as they deal with a world in which an increasing amount of information is presented in quantitative ways and more and more occupations and fields of study rely on mathematics.

Performance Expectation

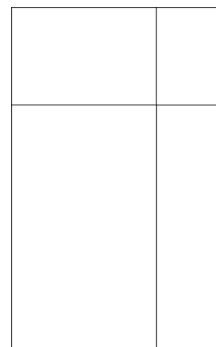
Students are expected to:

- A2.8.A Analyze a problem situation and represent it mathematically.
- A2.8.B Select and apply strategies to solve problems.
- A2.8.C Evaluate a solution for reasonableness, verify its accuracy, and interpret the solution in the context of the original problem.
- A2.8.D Generalize a solution strategy for a single problem to a class of related problems and apply a strategy for a class of related problems to solve specific problems.
- A2.8.E Read and interpret diagrams, graphs, and text containing the symbols, language, and conventions of mathematics.
- A2.8.F Summarize mathematical ideas with precision and efficiency for a given audience and purpose.

Explanatory Comments and Examples

Examples:

- Show that $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$, for all positive real values of a and b .
- Show that the product of two odd numbers is always odd.
- Leo is painting a picture on a canvas that measures 32 inches by 20 inches. He has divided the canvas into four different rectangles, as shown in the diagram.



He would like the upper right corner to be a rectangle that has a length 1.6 times its width. Leo wants the area of the larger rectangle in the lower left to be at least half the total area of the canvas.

Performance Expectation

Students are expected to:

- A2.8.G Use inductive reasoning and the properties of numbers to make conjectures, and use deductive reasoning to prove or disprove conjectures.
- A2.8.H Synthesize information to draw conclusions and evaluate the arguments and conclusions of others.

Explanatory Comments and Examples

Describe all the possibilities for the dimensions of the upper right rectangle to the nearest hundredth, and explain why the possibilities are valid.

If Leo uses the largest possible dimensions for the smaller rectangle:

- What will the dimensions of the larger rectangle be?
- Will the larger rectangle be similar to the rectangle in the upper right corner? Why or why not?
- Is the original canvas similar to the rectangle in the upper right corner?

(A rectangle whose length and width are in the ratio $\frac{1+\sqrt{5}}{2}$ (approximately equal to 1.6) is called a “golden rectangle” and is often used in art and architecture.)

- A relationship between variables can be represented with a table, a graph, an equation, or a description in words.
 - How can you decide from a table whether a relationship is linear, quadratic, or exponential?
 - How can you decide from a graph whether a relationship is linear, quadratic, or exponential?
 - How can you decide from an equation whether a relationship is linear, quadratic, or exponential?